Middle Grades Mathematics Graduation Competencies

All students graduate college, career, and life ready

Personalized Learning Tenets

Learner Profiles

Competency-Based Learning

Authentic Learning

21st Century Skills

Technology-enabled

Student Agency

School Autonomy/District Support
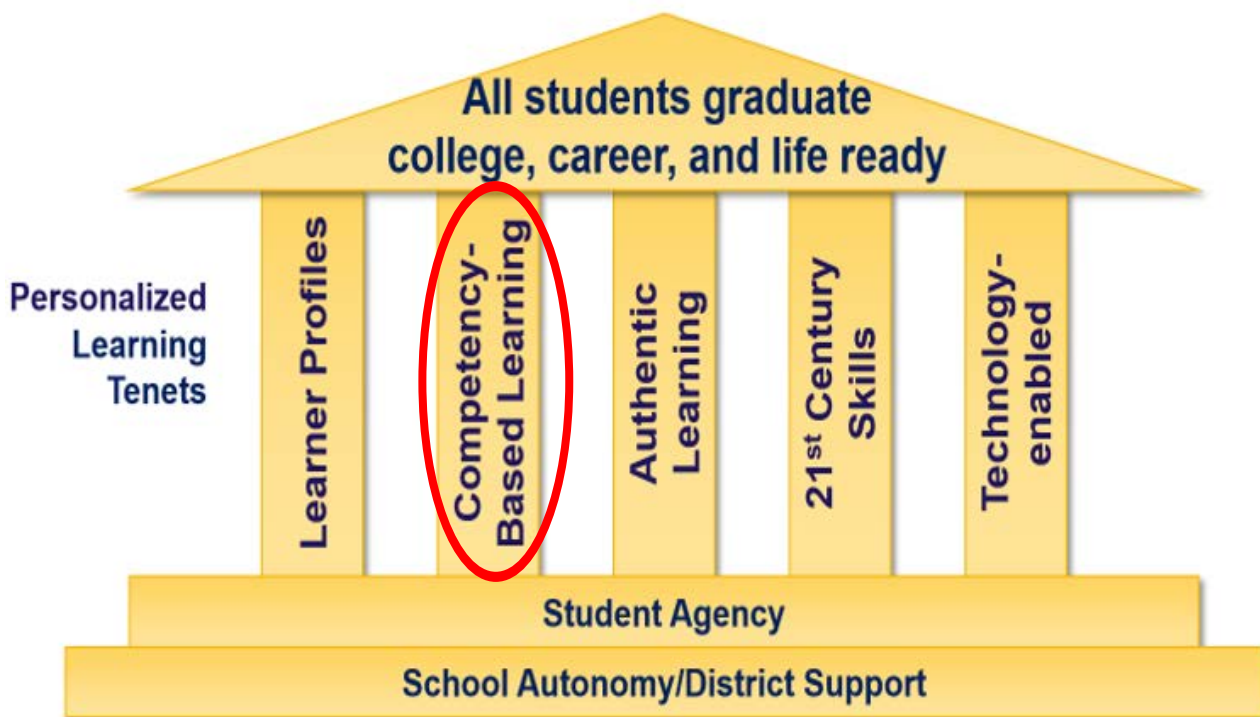
Graduation Competency 1. The student uses mathematical practices to help make sense of the real world. The student can identify variables, formulate a model describing a relationship between the variables, interpret results, and validate and report conclusions and the reasoning behind them.

Middle Grades Performance Indicators (6-8)
1. Students can make sense of problems and persevere in solving them.
2. Students can reason abstractly and quantitatively.
3. Students can construct viable arguments and critique the reasoning of others.
4. Students can model with mathematics.
5. Students can use appropriate tools strategically.
6. Students can attend to precision.
7. Students can look for and make use of structure.
8. Students can look for and express regularity in repeated reasoning.

Performance Indicator	Addressed in this competency	Grade Level	Courses
1. Students can make sense of problems and persevere in solving them.	● explaining to themselves the meaning of a problem and looking for entry points to its solution ● analyze givens, constraints, relationships, and goals make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt ● consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution ● monitor and evaluate their progress and change course if necessary (older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need). ● Explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. (younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem). ● check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" ● understand the approaches of others to solving complex problems and identify correspondences between different approaches.	K-12	All
2. Students can reason abstractly and quantitatively.	● make sense of quantities and their relationships in problem situations ● students have the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. ● create a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	K-12	All
	● understand and use stated assumptions, definitions, and previously	K-12	All

3. Students can construct viable arguments and critique the reasoning of others.	established results in constructing arguments ● make conjectures and build a logical progression of statements to explore the truth of their conjectures ● analyze situations by breaking them into cases, and can recognize and use counterexamples ● justify their conclusions, communicate them to others, and respond to the arguments of others ● reason inductively about data, making plausible arguments that take into account the context from which the data arose ● compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is ● construct arguments using concrete referents such as objects, drawings, diagrams, and actions ● listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments		
4. Students can model with mathematics.	● solve problems arising in everyday life, society, and the workplace ● comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later ● identify important quantities in a practical situation and map their relationships using uch tools as diagrams, two-way tables, graphs, flowcharts and formulas and analyze those relationships mathematically to draw conclusions ● routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose	K-12	All
5. Students can use appropriate tools strategically.	● consider the available tools when solving a mathematical problem (pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software) ● sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations ● detect possible errors by strategically using estimation and other mathematical knowledge ● when making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. ● identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems ● use technological tools to explore and deepen their understanding of concepts	K-12	All
6. Students can attend to precision.	● communicate precisely to others ● use clear definitions in discussion with others and in their own reasoning ● state the meaning of the symbols they choose, including using the equal sign consistently and appropriately ● careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem ● calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context	K-12	All

7 Students can look for and make use of structure.	● look closely to discern a pattern or structure ● recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems and can step back for an overview and shift perspective ● see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects	K-12	All
8. Students can look for and express regularity in repeated reasoning.	● notice if calculations are repeated, and look both for general methods and for shortcuts ● maintain oversight of the process, while attending to the details ● evaluate the reasonableness of their intermediate results	K-12	All

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Graduation Competency #2: **The student reasons, describes and analyzes quantitatively using units and number systems to make sense of and solve problems.** Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understand that these properties lead to fluency with operations.

Performance Indicators Middle (6-8) for Competency 2.
Ratios and Proportional Relationships a. Understand and analyze ratio concepts and use ratio reasoning to solve problems. **The Number System** b. Apply and extend previous understandings of multiplication and division to divide fractions by fractions. c. Compute fluently with multi-digit numbers and find common factors and multiples. d. Apply and extend previous understandings of numbers to the system of rational numbers. e. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. f. Know that there are numbers that are not rational, and approximate them by rational numbers.

Performance Indicators Scoring Criteria for Competency 2.

Performance Indicators Grades 6-8 (MP 2,4,5,6,7)	Emerging	Progressing	Competent	Exemplary
Understand and analyze ratio concepts and use ratio reasoning to solve problems. MGSE6.RP.1, MGSE6.RP.2, MGSE6.RP.3 a-d, MGSE7.RP.1, MGSE7.RP.2 a-d, MGSE7.RP.3	The student can recall procedures involving ratios with limited understanding.	The student can use number sense to solve problems involving ratios and proportional relationships.	The student can analyze ratio concepts and solve problems using ratios. The student can also use appropriate strategies fluently (accurately, flexibly, and efficiently).	The student can defend a mathematical model in an authentic context and apply existing knowledge to a situation that solves a problem in the community or across multiple disciplines.
b. Apply and extend previous understandings of multiplication and division to divide fractions by fractions. MGSE6.NS.1	The student can calculate the product and quotient of fractions.	The student can represent division of fractions using models/diagrams.	The student can use appropriate strategies fluently (accurately, flexibly, and efficiently) to reason, describe, and analyze problems involving division of fractions by fractions.	The student can defend a mathematical model in an authentic context and apply existing knowledge to a situation that solves a problem in the community or across multiple disciplines.
c. Compute fluently with multi-digit numbers and find common factors and multiples. MGSE6.NS.2, MGSE6.NS.4, MGSE6.RP.1	The student can calculate the sum, difference, product and quotient of multi-digit numbers.	The student can use appropriate strategies to compute the sum, difference, product and quotient with multi-digit numbers to find common factors or multiples.	The student can use appropriate strategies to compute fluently (accurately, flexibly, and efficiently) with multi-digit numbers and find common factors and multiples.	The student can defend a mathematical model in an authentic context and apply existing knowledge to a situation that solves a problem in the community or across multiple disciplines.
d. Apply and extend previous understandings of numbers to the system of rational numbers. MGSE6.NS.3, MGSE6.NS.5, MGSE6.NS.6a-c, MGSE6.NS.7a-d, MGSE6.NS.8	The student can represent and describe the system of rational numbers.	The student can give examples and non-examples of rational numbers in contexts.	The student can apply and extend previous understandings of numbers to the system of rational numbers in an authentic context.	The student can defend a mathematical model in an authentic context and apply existing knowledge to a situation that solves a problem in the community or across multiple disciplines.

e. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. MGSE6.NS.3, MGSE7.NS.1 a-d, MGSE7.NS.2 a-d, MGSE7.NS.3	The student can calculate the sums, differences, products or quotients of rational numbers.	The student can use appropriate strategies to compute the sums, differences, products and quotients of rational numbers.	The student can apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers in an authentic context using appropriate strategies.	The competent student can defend a mathematical model in an authentic context and apply existing knowledge to a situation that solves a problem in the community or across multiple disciplines.
f. Know that there are numbers that are not rational, and approximate them by rational numbers. MGSE8.NS.1, MGSE8.NS.2	The student can define irrational numbers.	The student can give examples and non-examples of irrational numbers.	The student knows that there are numbers that are not rational, and can approximate them by rational numbers.	The competent student can defend a mathematical model in an authentic context and apply existing knowledge to a situation that solves a problem in the community or across multiple disciplines.

Sample Tasks:
https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/692
https://www.illustrativemathematics.org/content-standards/6/NS/B/4/tasks/256
https://www.illustrativemathematics.org/content-standards/6/NS/B/4/tasks/255
https://www.illustrativemathematics.org/content-standards/6/RP/A/2/tasks/1175
https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/310
https://www.illustrativemathematics.org/content-standards/8/NS/A/tasks/338

Graduation Competency #3: The student creates, interprets, uses, and analyzes patterns of algebraic structures to make sense of problems. Pattern sense gives students a lens with which to understand trends and commonalities. Students recognize and represent mathematical relationships and analyze change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Competency #3. Performance Indicators Middle School (6-8)
Expressions and Equations
a. Apply and extend previous understandings of arithmetic to algebraic expressions.
b. Reason about and solve one-variable equations and inequalities.
c. Represent and analyze quantitative relationships between dependent and independent variables.
d. Use properties of operations to generate equivalent expressions.
e. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
f. Work with radicals and integer exponents.
g. Understand the connections between proportional relationships, lines, and linear equations.
h. Analyze and solve linear equations and pairs of simultaneous linear equations.
Ratios and Proportional Relationships
i. Analyze proportional relationships and use them to solve real-world and mathematical problems.

Performance Indicators Scoring Criteria for Competency 3.

Performance Indicators Grades 6-8 (MP 2,4,5,6,7)	Emerging	Progressing	Competent	Exemplary
a. Apply and extend previous understandings of arithmetic to algebraic expressions. MGSE6.EE.2a-b, MGSE7.EE.1, MGSE7.EE.2, MGSE7.EE.4a	The student can define algebraic expression.	The student can identify and interpret patterns.	The student can apply and extend previous understandings of arithmetic to explain patterns by writing algebraic expressions.	The student can use algebraic patterns, structures, and proportional relationships to defend a mathematical model and/or problem that exists across disciplines.
b. Reason about and solve one-variable equations and inequalities. MGSE6.EE.5, MGSE7.EE.4, MGSE8.EE.7	The student can use rote procedures to solve one-variable equations or inequalities with limited understanding.	The student can explain solutions to routine problems involving one-variable equations and inequalities.	The student can analyze similarities/differences between procedures or solutions and solve one-variable equations and inequalities in an authentic context.	The student can use algebraic patterns, structures, and proportional relationships to defend a mathematical model and/or problem that exists across disciplines.
c. Represent and analyze quantitative relationships between dependent and independent variables. MGSE6.EE.6, MGSE6.EE.9a-b, MGSE7.EE.4, MGSE8.EE.5, MGSE8.EE.6	The student can use rote procedures to solve algebraic problems with limited understanding.	The student can explain solutions to routine problems involving dependent and independent variables.	The student can represent and analyze quantitative relationships between dependent and independent variables in an authentic context.	The student can use algebraic patterns, structures, and proportional relationships to defend a mathematical model and/or problem that exists across disciplines.
d. Use properties of operations to generate equivalent expressions. MGSE6.EE.3, MGSE6.EE.4, MGSE7.EE.3, MGSE8.EE.1	The student can define equivalent expressions.	The student can identify equivalent expressions.	The student can use properties of operations to generate equivalent expressions.	The student can use properties of operations to defend a mathematical model and/or problem that exists across disciplines.
e. Solve real-life and mathematical problems using numerical and algebraic expressions and equations. MGSE6.EE.2c, MGSE7.EE.4a & c, MGSE8.EE.8	The student can use rote procedures to solve algebraic problems with limited understanding.	The student can use appropriate strategies to solve algebraic expressions and equations.	The student can solve real-life and mathematical problems involving numerical and algebraic expressions and equations.	The student can use algebraic patterns, structures, and proportional relationships to defend a mathematical model and/or problem that exists across disciplines.

f. Solve problems with radicals and/or integer exponents. MGSE6.EE.1, MGSE8.EE.1, MGSE8.EE.2, MGSE8.EE.3, MGSE8.EE.4	The student uses rote procedures to solve algebraic problems involving radicals and integer exponents with limited understanding.	The student can use appropriate strategies to solve algebraic problems involving radicals and integer exponents.	The student can solve problems with radicals and integer exponents in real world problem situations.	The student can use algebraic patterns, structures, and proportional relationships to defend a mathematical model and/or problem that exists across disciplines.
g. Understand the connections between proportional relationships, lines, and linear equations. MGSE7.RP.2, MGSE7.RP.2a, MGSE8.EE.7	The student can identify proportional relationships, lines, or linear equations.	The student can identify proportional relationships, lines, and linear equations.	The student can explain the connections between proportional relationships, lines, and linear equations using multiple representations.	The student can use algebraic patterns, structures, and proportional relationships to defend a mathematical model and/or problem that exists across disciplines.
h. Analyze and solve linear equations and pairs of simultaneous linear equations. MGSE7.RP.2c, MGSE8.EE.8	The student can use rote procedures to solve linear equations with limited understanding.	The student can solve linear equations and pairs of simultaneous linear equations.	The student can analyze and solve linear equations and pairs of simultaneous linear equations using multiple strategies.	The competent student can use algebraic patterns, structures, and proportional relationships to defend a mathematical model and/or problem that exists across disciplines.
i.Analyze proportional relationships and use them to solve real-world and mathematical problems. MGSE6.RP.3, MGSE7.RP.2a, MGSE8.EE.8c	The student can recognize mathematical patterns and proportional relationships.	The student can identify patterns as proportional relationships.	The student can analyze proportional relationships and use them to solve real-world and mathematical problems.	The competent student can use algebraic patterns, structures, and proportional relationships to defend a mathematical model and/or problem that exists across disciplines.

Sample Tasks:
Expressions and Equations
http://www.insidemathematics.org/assets/problems-of-the-month/growing%20staircases.pdf
https://www.illustrativemathematics.org/content-standards/8/EE/B/5/tasks/129
https://www.illustrativemathematics.org/content-standards/8/EE/C/8/tasks/554
https://www.illustrativemathematics.org/content-standards/8/EE/A/3/tasks/476

Graduation Competency #4: The student uses functions to interpret and analyze a variety of contexts. Functions describe situations where one quantity determines another. Functions describe situations where one quantity determines another. In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. Functions presented as expressions can model many important phenomena.

Competency #4. Performance Indicators Middle School (6-8)
Functions   a. The student can define, evaluate, and compare functions.   b. The student can use functions to model relationships between quantities.

Performance Indicators Scoring Criteria for Competency 4

Performance Indicators Grades 6-8	Emerging	Progressing	Competent	Exemplary
a,The student can define, evaluate, and compare functions. MGSE8.F.1, MGSE8.F.2, MGSE8.F.3	The student can define functions.	The student can define and evaluate functions.	The student can define, evaluate, and compare functions.	The student can defend a mathematical model in an authentic context and apply existing knowledge to a situation involving functions and relationships between functions that solves a problem in the community or across multiple disciplines.
b. The student can use functions to model relationships between quantities. MGSE6.RP.3, MGSE7.RP.2, MGSE8.F.4, MGSE8.F.5	The student can identify inputs and outputs.	The student can use functions to model relationships between quantities, using a graph *or* equation.	The student can use functions to model relationships between quantities, using a graph *and* an equation.	The student can defend a mathematical model in an authentic context and apply existing knowledge to a situation involving functions and relationships between functions that solves a problem in the community or across multiple disciplines.
Sample Tasks: https://www.illustrativemathematics.org/content-standards/8/F/A/3/tasks/813				

Graduation Competency #5: The student proves, understands, and models geometric concepts using appropriate tools, theorems and constructions to solve problems and apply logical reasoning. Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Competency # 5. Performance Indicators Middle School (6-8)
Geometry   a. The student can solve real-world and mathematical problems involving angle measure, area, surface area, and volume.   b. The student can draw, construct and describe geometrical figures and describe the relationships between them.   c. The students can understand congruence and similarity using physical models, transparencies, or geometry software.   d. The student can understand and apply the Pythagorean Theorem.   e. The student can solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Performance Indicators Scoring Criteria for Competency 5

Performance Indicators Grades 6-8 (MP 2,4,5,6,7)	Emerging	Progressing	Competent	Exemplary
a, The student can solve real-world and mathematical problems involving angle measure, area, surface area, and volume. MGSE6.G.1, MGSE6.G.2, MGSE6.G.3, MGSE6.G.4, MGSE7.G.2, MGSE7.G.4, MGSE7.G.5, MGSE7.G.6, MGSE8.G.5	The student can identify the dimensions of geometric figures.	The student can solve mathematical problems using a formula or equation for angle measure, area, surface area, and volume.	The student can solve real-world and mathematical problems involving angle measure, area, surface area, and volume.	The student can justify or defend mathematical problems involving area, surface area, and volume.
b. The students can draw, construct and describe geometrical figures and describe the relationships between them. MGSE6.G.3, MGSE6.G.4, MGSE7.G.1, MGSE7.G.2, MGSE7.G.3, MGSE8.G.3	The student can draw and describe geometric figures.	The student can draw, construct and describe geometrical figures.	The student can draw, construct and describe geometrical figures and describe the relationships between them.	The student can defend a mathematical model by drawing, constructing and describing geometric figures in an authentic context that exists across disciplines.
c. Understand congruence and similarity using physical models,	The student can identify congruent *or* similar figures.	The student can identify congruent *and* similar figures.	The student can understand congruence and similarity using physical models,	The student can prove congruence and similarity using physical models,

			transparencies, or geometry software.	transparencies, or geometry software.
transparencies, or geometry software. MGSE8.G.1, MGSE8.G.2, MGSE8.G.3, MGSE8.G.4				
d. Understand and apply the Pythagorean Theorem. MGSE8.G.6, MGSE8.G.7, MGSE8.G.8	The student can identify the Pythagorean Theorem.	The student solves problems using the Pythagorean Theorem.	The student can understand and apply the Pythagorean Theorem to solve problems involving right triangles.	The student can create or justify a mathematical model to prove the Pythagorean Theorem.
e. Solve real-world and mathematical problems involving volume of cylinders, cones and spheres. MGSE8.G.9	Students can identify the formulas for the volume of cylinders, cones, and spheres.	The student can solve mathematical problems involving volume of cylinders, cones or spheres, using a formula.	The student can solve real-world and mathematical problems involving volume of cylinders, cones and spheres.	The student can design real-world problems involving volume of cylinders, cones and spheres that exist across disciplines.

Sample Tasks:
https://www.illustrativemathematics.org/content-standards/8/G/C/9/tasks/520
https://www.illustrativemathematics.org/content-standards/8/G/A/4/tasks/1946
http://www.insidemathematics.org/assets/problems-of-the-month/what's%20your%20angle.pdf

Graduation Competency #6: The student uses a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. The use of data and probability provide students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Probability provides the foundation for collecting, describing, and interpreting data.

Competency #6. Performance Indicators Middle School (6-8)
Statistics and Probability
a. The students can develop understanding of statistical variability.
b. The student can summarize and describe distributions.
c. The student can use random sampling to draw inferences about a population.
d. The students can draw informal comparative inferences about two populations.
e. The student can investigate chance processes and develop, use, and evaluate probability models.
f. The student can investigate patterns of association in bivariate data.

Performance Indicators Scoring Criteria for Competency 6

Performance Indicator Grades 6-8	**Emerging**	**Progressing**	**Competent**	**Exemplary**
a. The students can develop understanding of statistical variability. MGSE6.SP.1, MGSE6.SP.3, MGSE6.SP.4	The student can define variability.	The student can calculate statistical variability.	The student develops an understanding of statistical variability.	The student applies the concept of statistical variability to make inferences.

b. The student can summarize and describe distributions. MGSE6.SP.2, MGSE6.SP.3, MGSE6.SP.4, MGSE6.SP.5	The student can recognize distributions.	The student can summarize *or* describe distributions.	The student can summarize *and* describe distributions.	The student can analyze and synthesize distributions from multiple sources.
c. The student can use random sampling to draw inferences about a population. MGSE7.SP.1, MGSE7.SP.2	The student can conduct a random sample of a population.	The student can organize data from a random sample of a population.	The student can use random sampling to draw inferences about a population.	The student can analyze and synthesize information from multiple random samplings of a population.
d. The students can draw informal comparative inferences about two populations. MGSE7.SP.3, MGSE7.SP.4)	The student can compare information about two populations.	The student can interpret comparative inferences about two populations.	The student can draw informal comparative inferences about two populations.	The student can design a situation in which they can draw informal comparative inferences about two populations.
e. The student can investigate chance processes and develop, use, and evaluate probability models. MGSE7.SP.5, MGSE7.SP.6, MGSE7.SP.7 a-b, MGSE7.SP.8	The student can identify situations involving chance.	The student can evaluate situations involving chance using probability models.	The student can investigate chance processes and develop, use, and evaluate probability models.	The student can analyze and synthesize information from multiple sources involving chance processes.
f. The student can investigate patterns of association in bivariate data. MGSE8.SP.1, MGSE8.SP.2, MGSE8.SP.3, MGSE8.SP.4	The student can define bivariate data.	The student can collect and display patterns of association in	The student can investigate patterns of association in bivariate data.	The student can analyze patterns of association in bivariate data.
Sample Tasks: https://www.illustrativemathematics.org/content-standards/7/SP/A/2/tasks/1339 https://www.illustrativemathematics.org/content-standards/8/SP/A/1/tasks/1097				